Fundamentals of Communications Engineering

Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
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Room: Comm-02

Lecture: 17

Angle Modulation

Angle Modulation

- **Frequency Modulation (FM)**
  Frequency of the carrier is varied according to the message signal amplitude.
  Digital format is FSK
- **Phase Modulation (PM).**
  Phase angle of the carrier is varied according to the message signal amplitude
  Digital format is PSK

* Angle Modulation used in

1. Radio broadcasting,
2. Two way mobile radio,
3. Microwave communication,
4. TV sound transmission,
5. Cellular radio, and
6. Satellite communication.

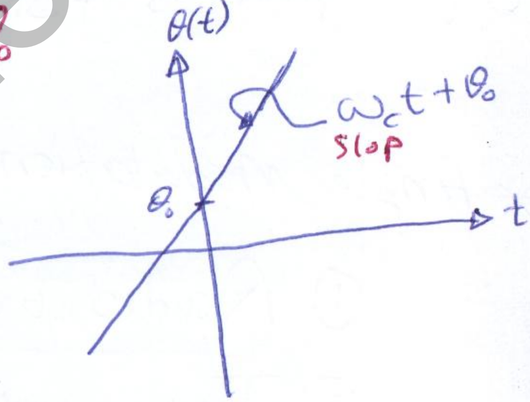
* Instantaneous Frequency :

+ consider a generalized sinusoidal signal $\varphi(t)$:

$$\varphi(t) = A\cos[\theta(t)] \text{ ——— } (5.1)$$

where $\theta(t)$ is the generalized angle, and it is a function of time t.

+ The generalized angle of conventional sinusoid $A\cos(\omega_c t + \theta_0)$ is $\omega_c t + \theta_0$



* $\omega_c t + \theta_0$ is a staight line with a slope ω_c and intercept θ_0

* Hence; $\varphi(t) = A\cos(\omega_c t + \theta_0)$

* The frequency of $\varphi(t)$ is the slope of $\theta(t)$

* Therefore, the instantaneous frequency at any time t is ω_i

$$\boxed{\omega_i(t) = \frac{d\theta}{dt}} \text{ — } (5.2a) \quad \text{and} \quad \boxed{\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\, d\alpha} \text{ — } (5.2b)$$

* A modulating signal (message signal) $m(t)$ is possible to transmit it by varying the angle θ of a carrier.

* Carrying $m(t)$ on the angle of the carrier is called angle modulation or exponential modulation.

* Two simple possibilities are FM & PM.

* In PM, the angle $\theta(t)$ is varied linearly with $m(t)$:-

$$\boxed{\theta(t) = \omega_c t + \theta_0 + k_p m(t)}$$

where k_p is a constant and ω_c is the carrier frequency

let $\theta_0 = 0$:-

$$\boxed{\theta(t) = \omega_c t + k_p m(t)} \quad (5.3a)$$

∴ The PM wave is ∴

$$\boxed{\varphi_{PM}(t) = A\cos\left[\omega_c t + k_p m(t)\right]} \quad (5.3b)$$

* The instantaneous frequency in PM thus:-

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + k_p \frac{d\,m(t)}{dt}$$

$$\omega_i(t) = \omega_c + k_p \dot{m}(t) \qquad (5.3c)$$

∴ In PM : the instantaneous frequency ω_i varies linearly with the derivative of $m(t)$

in PM $\quad \omega_i(t) \propto \dot{m}(t)$

* If ω_i varies linearly with $m(t)$ itself not with $\dot{m}(t)$, then we have FM modulation

* In FM : ω_i varies linearly with $m(t)$

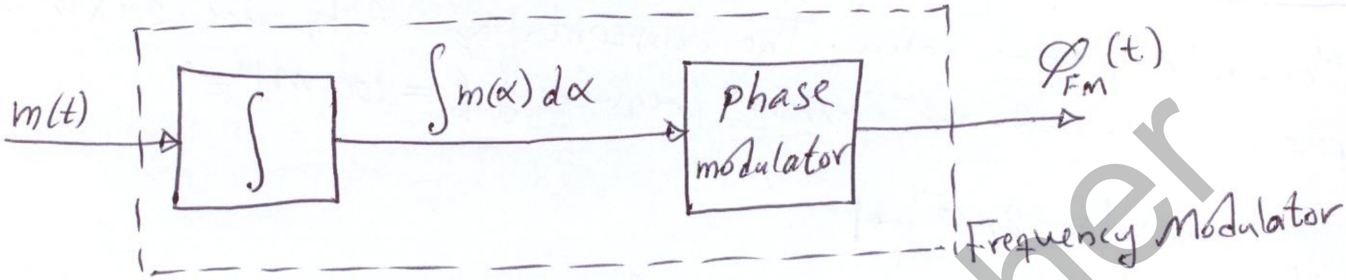
∴ $$\boxed{\omega_i(t) = \omega_c + k_f\, m(t)} \qquad (5.4a)$$

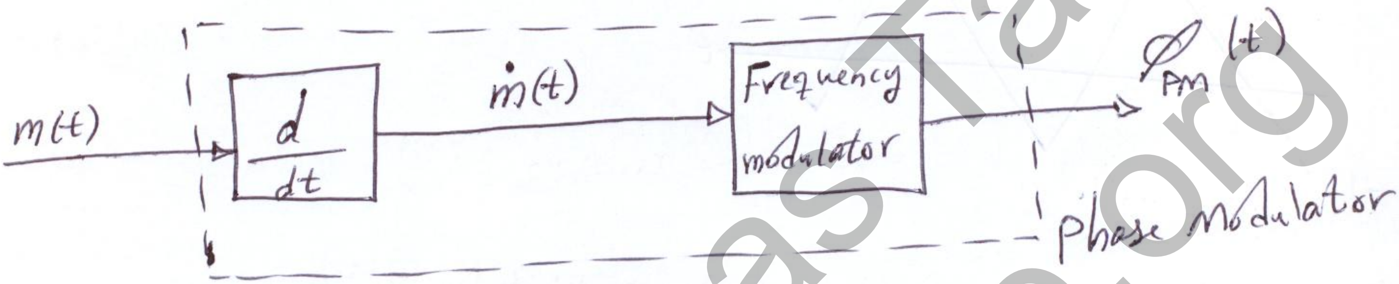
where k_f is a constant.

∴ $$\theta(t) = \int_{-\infty}^{t} [\omega_c + k_f\, m(\alpha)\, d\alpha] = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha \qquad (5.4b)$$

∴ FM wave is:

$$\boxed{\varphi_{FM}(t) = A \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]} \qquad (5.4c)$$





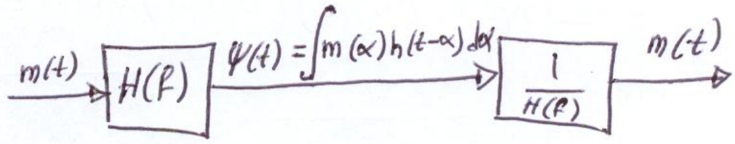
* The generalized angle-modulated carrier $\varphi_{EM}(t)$ can be:-

$$\varphi_{EM}(t) = A\cos(\omega_c t + \psi) \qquad (5.5a)$$

$$= A\cos\left[\omega_c t + \int_{-\infty}^{\infty} m(\alpha)h(t-\alpha)\,d\alpha\right] \qquad (5.5b)$$

* For PM:

$$h(t) = k_p\,\delta(t)$$

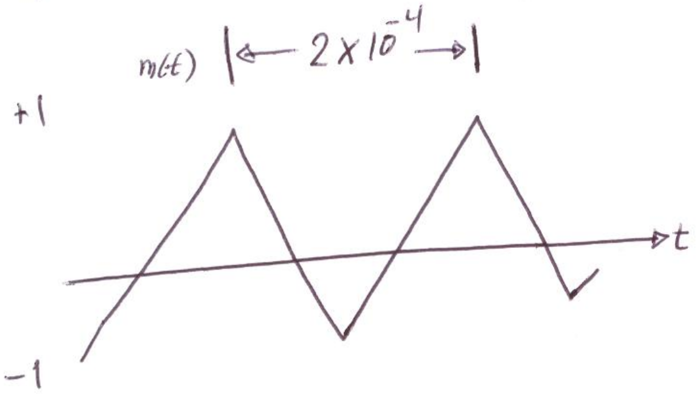


* For FM:

$$h(t) = k_f\,u(t)$$

* where by this generalization $h(t)$ can be any other function. Hence, milions of angle modulations can be produced.

EX. 5.1 Sketch FM and PM waves for the modulating signal $m(t)$ shown in Figure below. The constants k_f and k_p are $2\pi \times 10^5$ and 10π, respectively, and the carrier frequency $f_c = 100$ MHz.



Solution For FM:- $\omega_i = \omega_c + k_f\, m(t)$

or $f_i = f_c + \frac{k_f}{2\pi} m(t) = 10^8 + 10^5 m(t)$

* minimum f_i is when $m(t)$ is minimum.

minimum $m(t) = -1$

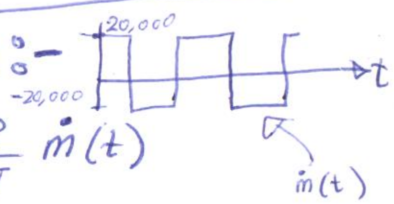
maximum $m(t) = +1$

$$(f_i)_{min} = 10^8 + 10^5 [m(t)]_{min}$$

$$f_{i_{min}} = 10^8 + 10^5(-1) = 99.9 \text{ MHz}.$$

$$f_{i_{max}} = 10^8 + 10^5(+1) = 100.1 \text{ MHz}.$$

For PM:-

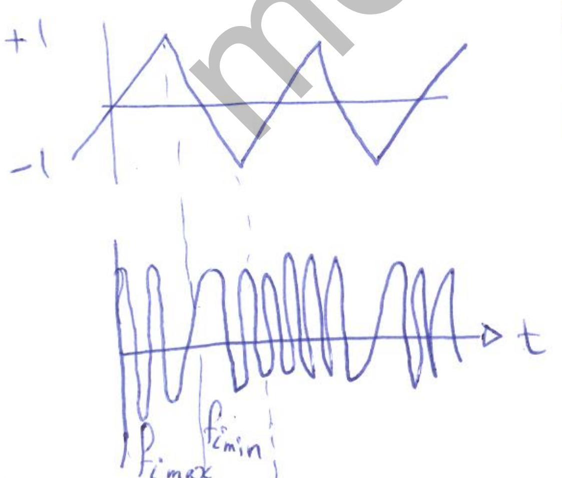


$$f_i = f_c + \frac{k_p}{2\pi} \dot{m}(t)$$

$$f_i = 10^8 + 5\, \dot{m}(t)$$

$$f_{i_{min}} = 10^8 + 5[\dot{m}(t)]_{min} = 10^8 + 5(-20{,}000) = 99.9 \text{ MHz}$$

$$f_{i_{max}} = 10^8 + 5[\dot{m}(t)]_{max} = 10^8 + 5(20{,}000) = 100.1 \text{ MHz}.$$



Power of Angle Modulated wave:

* In angle Modulation, the frequency or the phase of the carrier varies in proportion with the amplitude of the message signal (modulating signal).

* Hence, the amplitude of the carrier is constant.

∴ Power of FM or PM modulated wave is constant:

$$\boxed{P_{EM} = P_{FM} = P_{PM} = \frac{A^2}{2}}$$

$$\frac{(\text{Amplitude of carrier})^2}{2}$$

Bandwidth of Angle-Modulated wave :

* Let $a(t) = \int_{-\infty}^{t} m(\alpha)\, d\alpha$ ——— (5.6)

* Let $\hat{\varphi}_{FM}(t) = A e^{j[\omega_c t + k_f a(t)]} = A e^{jk_f a(t)} e^{j\omega_c t}$ ——— (5.7a)

** Now the real part is :

$$\varphi_{FM}(t) = R_e\, \hat{\varphi}_{FM}(t) \quad \text{——— (5.7b)}$$

Remember the power series of e^x is

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\therefore \hat{\varphi}_{FM}(t) = A\left[1 + jk_f a(t) - \frac{k_f^2}{2!} a^2(t) + \cdots + j^n \frac{k_f^n}{n!} a^n(t) + \cdots\right] e^{j\omega_c t} \quad \text{——— (5.8a)}$$

** Now the real part is :

$$\varphi_{FM}(t) = R_e[\hat{\varphi}_{FM}(t)] = A\Big[\underbrace{\cos\omega_c t}_{\text{carrier}} - \underbrace{k_f a(t) \sin\omega_c t}_{\text{DSB-SC}}$$

$$- \frac{k_f^2}{2!} \underbrace{a^2(t) \cos\omega_c t}_{\substack{\text{DSB-SC} \\ a^2(t)}} + \frac{k_f^3}{3!} \underbrace{a^3(t) \sin\omega_c t}_{\substack{\text{DSB-SC} \\ a^3(t)}} + \cdots\Big] \quad \text{——— (5.8b)}$$

* Since $a(t)$ = integral of $m(t)$:

* Since $M(\omega)$ band limited to B Hz :-

$$a^2(t) \xleftrightarrow{F.T.} A(\omega) * A(\omega)/2\pi$$

Bandwidth of $a^2(t) \leftrightarrow A(\omega) * \frac{A(\omega)}{2\pi}$ is 2B Hz.

→ Bandwidth of $a^n(t)$ is nB Hz centered around ω_c.

∴ Theoretically, FM Bandwidth $= \infty$.

Then there are two distinct possibilities in terms of bandwidth:

1. Narrowband FM, (NBFM)
2. wideband FM. (WBFM).

* Narrow-Band Angle Modulation :

* Angle modulation is non-linear unlike AM modulation.

* $A\cos\{\omega_c t + K_p[a_1(t) + a_2(t)]\} \neq A\cos[\omega_c t + K_p a_1(t)] + A\cos[\omega_c t + K_p a_2(t)]$

the principle of superposition does not hold. As we have studied previously.

* If $|K_f\, a(t)| \ll 1$ then Eq. 5.8 becomes :

$$\boxed{\phi_{FM}(t) \simeq A\left[\cos\omega_c t - K_f\, a(t)\sin\omega_c t\right]} \quad (5.9)$$

* This is a linear modulation similar to AM.

* In AM, the bandwidth $= 2B$ and in (5.9) $\phi_{FM}(t)$ has bandwidth $= 2B$. Thus,

NBFM is when $\boxed{|K_f\, a(t)| \ll 1}$

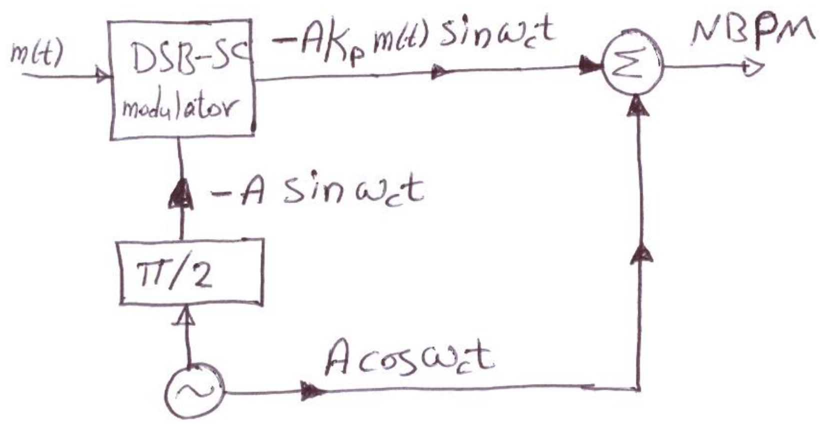
* Similarly, the PM also will be Narrow-band PM (NBPM) :

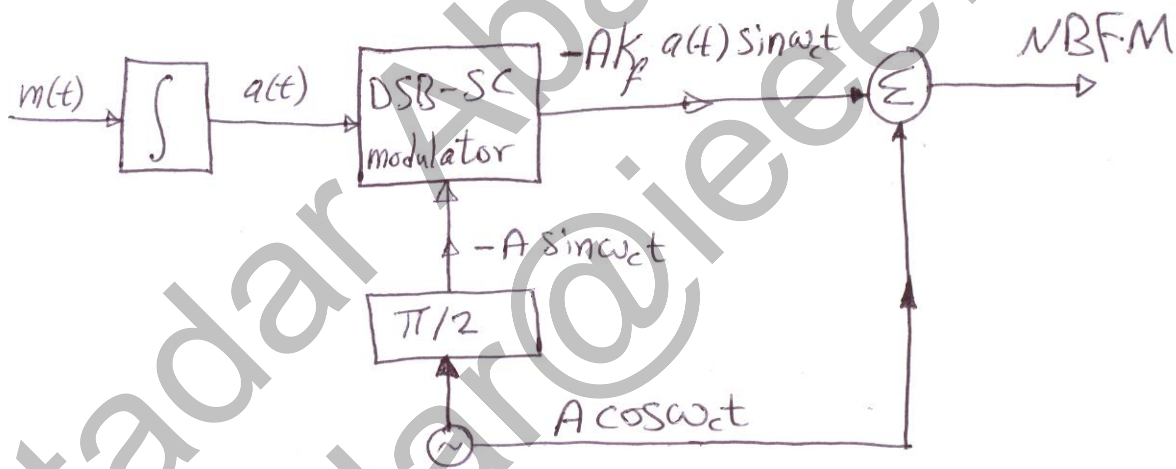
$$\boxed{\phi_{PM} \cong A\left[\cos\omega_c t - k_p\, m(t)\sin\omega_c t\right]} \quad (5.10)$$

* AM and NBFM both have pure carrier and sidebands centered at $\pm\omega_c$

* Both AM & NBFM bandwidths $= 2B$.

* The sideband spectrum For FM has phase shift $= \frac{\pi}{2}$ w.r.t. carrier. whereas that of AM is in phase with the carrier.


m(t)
DSB-SC modulator
-AKp m(t) sinωct
Σ
NBPM
-A sinωct
π/2
A cosωct


m(t)
∫
a(t)
DSB-SC modulator
-AKf a(t) sinωct
Σ
NBFM
-A sinωct
π/2
A cosωct